Name _____ Date 11

Using Powers and Square Roots

Power: An expression that has a base and an exponent. The **base** is a factor that is multiplied by itself the number of times named by the **exponent.**

base 5^{3} exponent

Square Number: The product of a number and itself. Also, a number with the exponent of two.

Square Root: One of two equal factors of a number.

$5^3 = 5 \times 5 \times 5 = 125$

$5^2 = 5 \times 5 = 25$

$\sqrt{25} = 5$ or $^-5$

$100^0 = 1, 5^0 = 1$

The $\sqrt{2}$ is an irrational number. Square and square root are inverse operations!

Write the value of each expression.

1. $1^{10} =$ 1
2. $4^3 =$ 64
3. $\sqrt{9} =$ 3
4. $2^4 =$ 16
5. $3^1 =$ 3
6. $\sqrt{16} =$ 4
7. $\sqrt{81} =$ 9
8. $\sqrt{36} =$ 6

Write each expression in words.

9. 6^7 six to the seventh power / six to the seventh / base six exponent seven
10. $\sqrt{16}$ root sixteen / square root sixteen
11. 3^2 three squared

20

Name _____

Squares and Square Roots

6

We say that any number raised to the power of 2 is "squared." The perfect squares are squares of whole numbers. Here are the first three perfect squares.

$1^2 = 1 \times 1 = 1$

$2^2 = 2 \times 2 = 4$

Any perfect square can be explained using a square!

$3^2 = 3 \times 3 = 9$

The square root of a number, *n*, is a number that when multiplied by itself, equals *n*. Here are the square roots of the perfect squares above.

$1^2 = 1 \quad \sqrt{1} = 1$
$2^2 = 4 \quad \sqrt{4} = 2$
$3^2 = 9 \quad \sqrt{9} = 3$

Solve.

1. $9^2 =$ 81 $\quad 7^2 =$ 49 $\quad 10^2 =$ 100 $\quad 3^2 =$ 9

2. $\sqrt{25} =$ 5 $\quad \sqrt{36} =$ 6 $\quad \sqrt{100} =$ 10 $\quad \sqrt{64} =$ 8

3. $12^2 =$ ~~24~~ 144 $\quad 8^2 =$ 64 $\quad 15^2 =$ 225 $\quad 20^2 =$ 400

4. $\sqrt{49} =$ 7 $\quad \sqrt{400} =$ 20 $\quad \sqrt{225} =$ 15 $\quad \sqrt{196} =$ 14

5. $\sqrt{625} =$ 25 $\quad 32^2 =$ 1024 $\quad \sqrt{2500} =$ 50 $\quad 125^2 =$ 15,625

20

Name _____

Exponents

7

Here are some other facts to remember about exponents:

The cube of a number is that number multiplied by itself three times.

$2^3 = 2 \times 2 \times 2 = 8$

To multiply exponential numbers with the same base, add the exponents.

$2^3 \times 2^2 = 2^{(3+2)} = 2^5$

$8 \times 4 = 32$

To divide exponential numbers with the same base, subtract the exponents.

$2^3 \div 2^2 = 2^{(3-2)} = 2^1$

$8 \div 4 = 2$

Solve.

1. $2^3 =$ 8 $\quad 4^3 =$ 64 $\quad 5^2 \div 5^1 =$ 5 $= 25 \div 5 = 5^{2-1}$ $\quad 6^2 \times 6^3 =$ $6^5 = 7776$

2. $8^4 \div 8^1 =$ $8^3 = 512$ $\quad 12^6 \div 12^5 =$ 12 $\quad 4^3 \times 4^2 =$ $4^5 = 1024$ $\quad 1^7 \div 1^5 =$ 1 $= 1^2$

3. $4^3 \div 4^2 =$ 4 $\quad 2^3 \div 2^1 =$ $2^2 = 4$ $\quad 8^2 \times 8^1 =$ $8^3 = 512$ $\quad 10^3 \times 10^2 =$ $10^5 = 100{,}000$

4. $6^2 \div 6^1 =$ 6 $\quad 3^5 \times 3^{-2} =$ $3^3 = 27$ $\quad 10^{-5} \times 10^3 =$ $10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$ $\quad 4^2 \times 4^{-1} =$ 4

5. $5^1 \div 5^1 =$ 1 $= 5^0$ $\quad 8^3 \times 8^{-1} =$ $8^2 = 64$ $\quad 0^{-3} \times 0^{-4} =$ 0 $\quad 7^4 \div 7^3 =$ 7

Name _____ Date _____

Shortcuts

18

Product Property: When multiplying powers having the same base, add the exponents.

$a^2 \times a^3 = a^{2+3} = a^5$

Power Property: When finding the power of a power, multiply the exponents.

$(a^2)^3 = a^{2\times3} = a^6$

Solve.

1. $a^6 \times a^3 =$ a^9
2. $(b^2)^4 =$ b^8
3. $y^5 \times y^7 =$ y^{12}
4. $(z^5)^5 =$ z^{25}
5. $(a^6)^4 =$ a^{24}
6. $b^5 \times b^2 =$ b^7
7. $a^3 \times a^{10} =$ a^{13}
8. $(y^5)^2 =$ y^{10}
9. $b^4 \times b^5 =$ b^9
10. $(a^2)^{10} =$ a^{20}
11. $a^{-4} \times a^5 =$ a
12. $(a^{-3})^{-3} =$ a^9
13. $a^2 \times a^{-2} =$ $a^0 = 1$
14. $y^2 \times y^3 =$ y^5

Without actually solving the expressions, determine the relationship of the following and write the correct symbol in the circle to make it true. Assume a is a positive integer (1, 2, 3 ----)

15. $a^3 \times a^{10}$ (a^{13}) $\geq$ $(a^3)^4$ (a^{12})
16. $a^5 \times a^1$ (a^6) $\geq$ $(a^1)^5$ (a^5)
17. $a^{-3} \times a^3$ ($a^0 = 1$) $\leq$ $(a^1)^1$ (a^1)
18. $a^0 \times a^{100}$ (a^{100}) $=$ $(a^{10})^{10}$ (a^{100})

15, 16, 17 could be "=" if a = 1

Name _____ Date 18

Division Property of Exponents

Division Property: When dividing powers having the same base, subtract the exponents.

$a^4 \div a^2 = a^{4-2} = a^2$ $\qquad \frac{a^4}{a^2} = a^{4-2} = a^2$

Simplify the expressions.

1. $a^6 \div a^3 = a^3$
2. $\frac{b^4}{b^2} = b^2$
3. $y^8 \div y^7 = y$
4. $\frac{b^6}{b^2} = b^4$
5. $\frac{y^5}{y^3} = y^2$
6. $z^5 \div z^2 = z^3$
7. $a^{10} \div a^3 = a^7$
8. $\frac{y^7}{y^3} = y^4$
9. $z^{125} \div z^{119} = z^6$
10. $\frac{b^{257}}{b^{236}} = b^{21}$
11. $a^{576} \div a^{572} = a^4$
12. $\frac{b^2}{b^2} = b^0 = 1$
13. $a^2 \div a^{-2} = a^4$
14. $a^2 \div y^{-3} = a^2 \div y^{-3}$ (cannot simplify without knowing the values of a & y)

Simplify the expression. Then find the answer.

15. $2^{99} \div 2^{95} = 2^4 = 16$
16. $3^{57} \div 3^{56} = 3$
17. $765^{50} \div 765^{50} = 765^0 = 1$
18. $3^2 \div 2^1 = 9 \div 2 = \frac{9}{2} = 4.5$

15

Name _____

63 Order of Operations

Remember the order of operations.

1. If there are parentheses, work inside them first.
2. Simplify exponents.
3. Multiply, and then divide from left to right.
4. Add and subtract from left to right.

$24 \div 6(4-2)+2^3 =$
$24 \div 6(2)+2^3 =$
$24 \div 6(2)+8 =$
$24 \div 12+8 =$
$2+8=10$

Solve. Show your work.

1. $5 \times 7+3 =$ $35+3=38$

2. $12-7 \times 2 =$ $12-14=-2$

3. $15 \div 3+4 =$ $5+4=9$

4. $8 \times 6+5 =$ $48+5=53$

5. $20-10 \times 3 =$ $20-30=-10$

6. $25 \div 5+14 =$ $5+14=19$

7. $9 \times 2+8 =$ $18+8=26$

$3^3 \times 2-16 \div 4 =$ $9 \times 2-16 \div 4$
$= 18-4=14$

$4^3 \div 8+18 \times 2 =$ $64 \div 8+18 \times 2$
$= 8+36=44$

$5^3 \times 4-20 \div 5 =$ $125 \times 4-20 \div 5$
$= 500-4=496$

$6^3 \div 8+22 \times 4 =$ $216 \div 8+22 \times 4$
$= 27+88=115$

$7^3 \times 6-24 \div 6 =$ $343 \times 6-24 \div 6$
$= 2058-4=2054$

$8^3 \div 2+26 \times 3 =$ $512 \div 2+26 \times 3$
$= 256+78=334$

$9^3 \times 8-27 \div 3 =$ $729 \times 8-27 \div 3$
$= 5832-9=5823$

8. Which has a greater value: $23 \times 2-12 \div 3$ or $24-9 \div 3(2+3)$?

$= 46-4$
$= 42$

$= 24-9 \div 3(5)$
$= 24-3 \cdot 5$
$= 24-15$
$= 9$

8

Name _____

Order of Operations

65

To evaluate an expression requiring several operations, follow this sequence:

- Do operations in **P**arentheses and other grouping symbols first. If there are grouping symbols within other grouping symbols, do the innermost first.
- Convert **E**xponential numbers to standard numbers.
- **M**ultiply and **D**ivide from left to right.
- **A**dd and **S**ubtract from left to right.

$12 \times 3 - 2(8+4) + 3^2 = ?$

Think of the acronym **PEMDAS** to remember the order of operations.

$12 \times 3 - 2(8 + 4) + 3^2 = ?$

Parentheses

$12 \times 3 - 2(12) + 3^2 = ?$

Exponents

$12 \times 3 - 2(12) + 9 = ?$

Multiply and Divide

$36 - 24 + 9 = ?$

Add and Subtract

$36 - 24 + 9 = 21$

Evaluate these expressions.

1. $2^2 + 2(3+4) \div 9 = 2^2 + 2(7) \div 9 = 4 + 2(7) \div 9 = 4 + 14 \div 9 = 4 + \frac{14}{9}$
2. $(16 \div 4) + (14 \div 2) - 3^2 = (4) + (7) - 3^2 = 4 + 7 - 9 = 2$
3. $(3^{-1} \times 9)^2 = (3^{-1} \times 3^2)^2 = (3^1)^2 = 3^2 = 9$
4. $20 \div (2+3) + 7 = 20 \div 5 + 7 = 4 + 7 = 11$
5. $7(8-5) + (4-1)^2 = 7(3) + (3)^2 = 7(3) + 9 = 21 + 9 = 30$
6. $(6+8) \times (6-8) = 14 \times (-2) = -28$
7. $(5 \times 4) + (2 \div 2) \times (7-4) = 20 + 1 \times 3 = 20 + 3 = 23$
8. $2(3)^2 + 5(4) - 6 = 2(9) + 5(4) - 6 = 18 + 20 - 6 = 32$

5

Name _____

Order of Operations

66

Which expression has the greater value? Use < or >.

1. $(5+3)\times 4+4^2 \;(>)\; 5+(3\times 4)+4^2$ see facing page
2. $2\times(1+3^2)-6+8 \;(>)\; 2\times 1+(3^2-6)+8$
3. $(5^2\times 2)+(6^2\div 12) \;(<)\; 5^2\times(2+6^2)\div 12$
4. $(24\div 3)\times 2^2+9 \;(>)\; 24\div(3\times 2^2)+9$

Follow the order of operations and find the missing number.

5. $(3^2+?)\div 6+4=8$
6. $(2^2+?)\times(2^2-1)=15$
7. $(2^2+?)\div(2^2-1)=15$
8. $14-(12\div 2^2)+?=2$
9. $(4^2-7)^? = \frac{1}{9}$

Add parentheses, operation signs, and exponents where they belong.

10. $(7\times 6)+(10\times 2)=62$

$= 42 + 20$

$= 62$

Name _____ Date _____

Order of Operations

12

Order of Operations: A mathematical procedure to evaluate an expression involving more than one mathematical operation.

The Order of Operations

1. First, evaluate any operations in PARENTHESES or brackets.
2. Second, evaluate any EXPONENTS.
3. Third, evaluate any MULTIPLICATION and DIVISION in order from left to right. If both operations are in an equation, evaluate whichever comes first from left to right.
4. Finally, evaluate any ADDITION and SUBTRACTION in order from left to right.

Evaluate the following expressions. See facing page

1. $[(6 + 7) + 3^2] \times (4 \div 8) =$ 11
2. $(2 + 3)^2 + (3 + 1)^2 =$ 41
3. $9 + 6 \div 3 \times 6 =$ 21
4. $7 + 14 - 3 + 5 \times 2^3 =$ 58
5. $11 + 3^2 - 4 \times (80 \div 10) =$ -12
6. $3^2 + 4 \times 3 \div 6 =$ 11
7. $[4 \times (5 + 6)] \div (4^2 - 12) =$ 11
8. $[(5 + 6) \times (9 - 2)] \div (2^4 - 5) =$ 7
9. $[4 + 7 \times (^-1 + 10)] - [^-3^2 - (^-8)] =$ 68
10. $[(^-10 \div 2) \times (^-8 - 3)] - (2^2 \times 2) =$ 47
11. $^-1 \times [^-4 \times (5 + 6)] \div (^-1) \times (5^0) =$ -44
12. $(5^3) \times (9 - 2) \div (2^4 \div 2^2) =$ 218 R3

80

Name _____ Date 4

Take the Challenge

Fill in the blank with a number that will make the expression correct.

1. $8 + \_\_\_\_ \div 4 = 12$
2. $6 + \_\_\_\_^2 \div 3 = 9$
3. $2 \times \_\_\_\_ + 5 - (1 \times 2) = 11$
4. $9 + 6 \div \_\_\_\_ \times 6 = 21$
5. $7 + 14 - 3 + 5 \times 2 \times \_\_\_\_ = 58$
6. $\_\_\_\_^2 + 4 \times 3 \div 6 = 18$

Insert parentheses where necessary to make the expression true.

7. $7 + 3 \div 5 \times 2 + 3 = 7$
8. $7 + 3 \div 5 \times 2 + 3 = 10$
9. $7 + 3 \div 5 \times 2 + 3 = 11.2$
10. $2 + 3^2 + 3 + 1^2 = 41$
11. $2 + 3^2 + 3 + 1^2 = 15$
12. $2 + 3^2 + 3 + 1^2 = 27$

Evaluate each expression and state if it is *true* or *false*. If it is false, state the correct answer.

13. $6 + (4 - 2) \times 3^2 \div 6 = 7$ See facing page False
    $= 9$
14. $6^2 + (3 - 1)^2 \times 3 \div 8 = 6$ False
    $= 37.5$
15. $[(9 + 7) \div 2] - 3 = 5$ True
16. $(10 \times [(8 + 6) - (5 + 5)] \div 2)^2 = 10$ False
    $= 400$